RADIATION PHYSICS NOTE NUMBER 22

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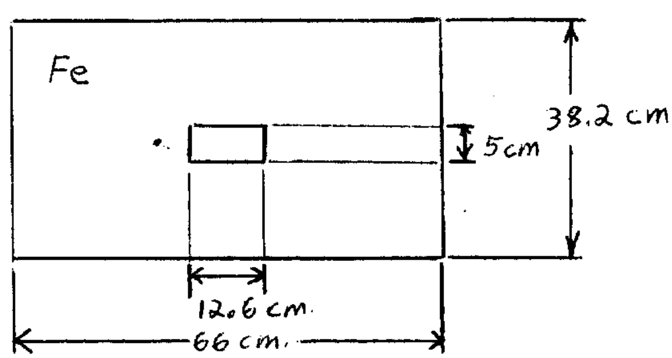
MAGNET ACTIVATION CALCULATION

Magnets at Fermilab are typically surveyed both internally with an instrument such as a Teletector and externally with an Elron. Readings are recorded in units of mrad/hr. From these readings it is desirable for a variety of reasons to be able to estimate the total activity in the magnet. This note presents the result of a calculation to accomplish this.

Case I: External Survey of a Typical Main Ring Magnet (610 cm long)

Such a magnet has a cross section as shown below (all dimensions in cm):



Here all material in the magnet is assumed to be iron. Surveys are typically taken at a distance of 1 ft (30 cm) from the outside of the magnet. One can then obtain an average reading in mrad/hr.

Several assumptions about the activation must now be made:

a) The activation is caused by random loss of beam in the interior of the magnet.

b) The stars produced by the hadronic cascade result in a γ ray decay spectrum at each star which is independent of its position within the magnet.

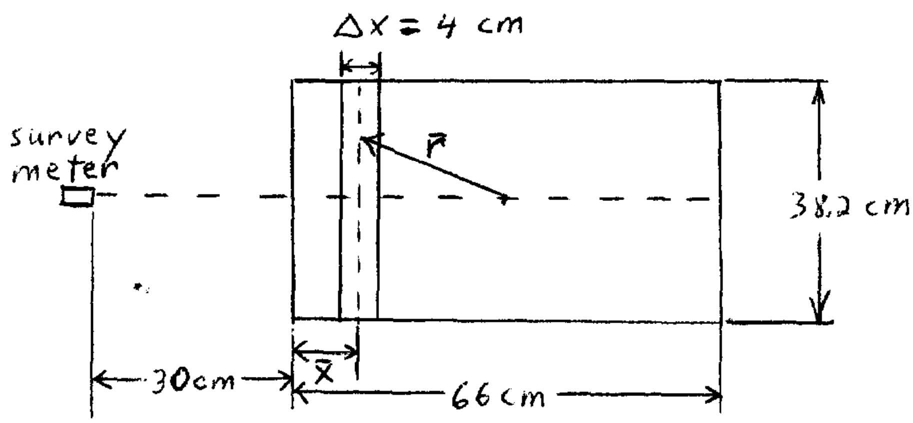
c) The average energy of the primary photons emitted is assumed to be 1 MeV. This follows from a simple average of the gamma rays emitted by activity produced in iron.[1]

d) The average energy of the photons measured externally is 0.73 MeV.[2]

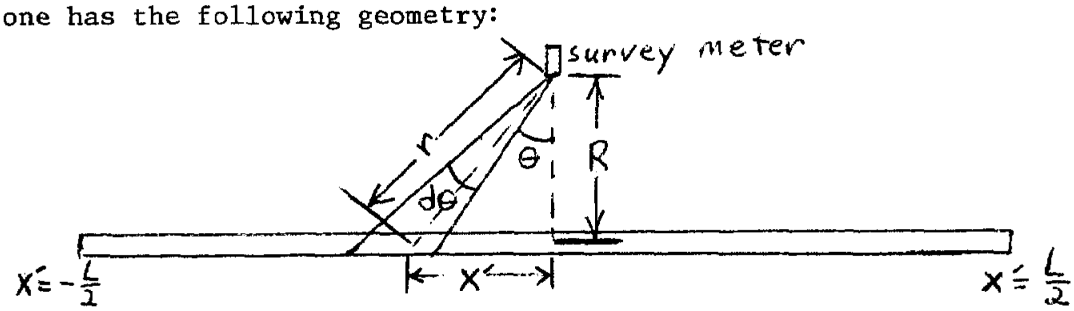
e) The distribution of stars in the magnet is obtained from the work of A. Van Ginneken and M. Awschalom.[3]

The general procedure of the calculation is to use the results of Ref. 3 to determine the distribution of stars in the magnet and then to calculate the reduced number of stars seen by the detector because of the effects of the shielding and geometry.

A numerical integration is now required, the geometry of which is shown below:



The magnet is divided into longitudinal slices 4 cm thick. The star density in each slice is taken to be that at $\bar{r}(\bar{x}) = \sqrt{(9.55)^2+(33-\bar{x})^2}$. The radiation from each slice may be considered to be due to a line source of length L. If, for example, the survey meter is centered longitudinally along the magnet, one has the following geometry:



$$\theta_{max} = \mathrm{Tan}^{-1}\frac{L}{2R} \qquad \text{(here L = 610 cm for a main ring bending magnet)}$$

$$dx' = \frac{r d\theta}{\cos\theta}$$

$$r\cos\theta = R$$

If the slice has a total source strength S in units of photons/sec, the flux/(sec cm^2) F at distance R is to be calculated. If instead S were a point source, at distance R one would have

$$F = \frac{S}{4\pi R^2} \tag{1}$$

For such a line source as the above we have at $d\theta$ a source of strength

$$\frac{S(R)}{L}\frac{r\ d\theta}{\cos\theta}$$

(Assuming a uniform spatial distribution which is approximately valid since the <u>average</u> reading is to be used and exactly valid of the beam is randomly lost in the pipe.)

$$F = \int_{-\theta_{max}}^{\theta_{max}} \frac{S(R)}{4\pi L}\frac{r d\theta}{r^2\cos\theta} = \frac{S(R)}{4\pi LR}\int_{-\theta_{max}}^{\theta_{max}} d\theta = \frac{S(R)}{2\pi LR}\theta_{max} \tag{2}$$

for R = 30 cm $\quad F = 0.234\ \frac{S(R)}{LR}$

for R = 96 cm $\quad F = 0.201\ \frac{S(R)}{LR}$

For simplicity the former factor will be used. Putting in buildup and attenuation factors, for each slice we have:

$$F = 0.234\frac{S(R)B(\mu\bar{x})e^{-\mu\bar{x}}}{L(\bar{x}+30)} = 3.836\text{x}10^{-4}\,\frac{S(R)B(\mu\bar{x})e^{-\mu\bar{x}}}{\bar{x}+30} \tag{3}$$

Thus by summing over all the slices, the ratio of observed flux/(sec$\cdot$cm^2) to total source strength can be determined. The quantities S(R) were taken from Ref. 3 while the values of $\bar{\mu}$ and $B(\mu\bar{x})$ were taken from Ref. 4. Doing the sum, the ratio of the total activity to the flux/(cm$^2\cdot$ sec) was found to be 1.06x10^7. This value was checked by taking a smaller slice thickness increment in ΔX and by checking the effect of the center open bore, effects which have been neglected up to this point. Neither effect exceeded 10%. Table I is a table of values used in the computation of this number. In Table I the strength of the activity in each slice is entered as the star density proton times 1000 at a depth of 100 cm. If a different depth is chosen approximately the same ratio is obtained because the decrease with radius is, to good approximation, exponential.

An effect not yet considered in the above treatment is that the consideration of the slice as a line source neglects the shielding effects due to the finite thickness of the slice itself for photons approaching the survey meter at nonzero values of θ. Since from Table I the slice centered at $\bar{x} = 2$ dominates the measurement with the survey meter, the calculation can be corrected by determining the magnitude of this effect for this slice alone.

At angle θ the mean thickness of the shield penetrated by photon is $\bar{t} = \frac{\mu\bar{x}}{\cos\theta}$ so that instead of eq. (3) one has:

$$F = \int_{-\theta_{max}}^{\theta_{max}} d\theta \, \frac{S}{4\pi LR} B(\bar{t}) e^{-\bar{t}}$$

or

$$F = \frac{S}{4\pi LR} \int_{-\theta_{max}}^{\theta_{max}} d\theta \, B(\bar{t}) e^{-\bar{t}} \tag{4}$$

Considering just the integral in eq. (4), setting $\bar{t} = \mu\bar{x}$ as was done in obtaining eq. (3):

$$\int_{-\theta_{max}}^{\theta max} d\theta \, B(\mu\bar{x}) e^{-\mu\bar{x}} = 2B(\mu\bar{x}) e^{-\mu\bar{x}} \theta_{max}$$
$$= 2.93 x 1.9 x 0.35 = 1.95$$

The integral in eq. (4) best lends itself to an approximate numerical technique. An integration step size of 0.2 radians was used. A table of values is given in Table II for the terms of the following sum:

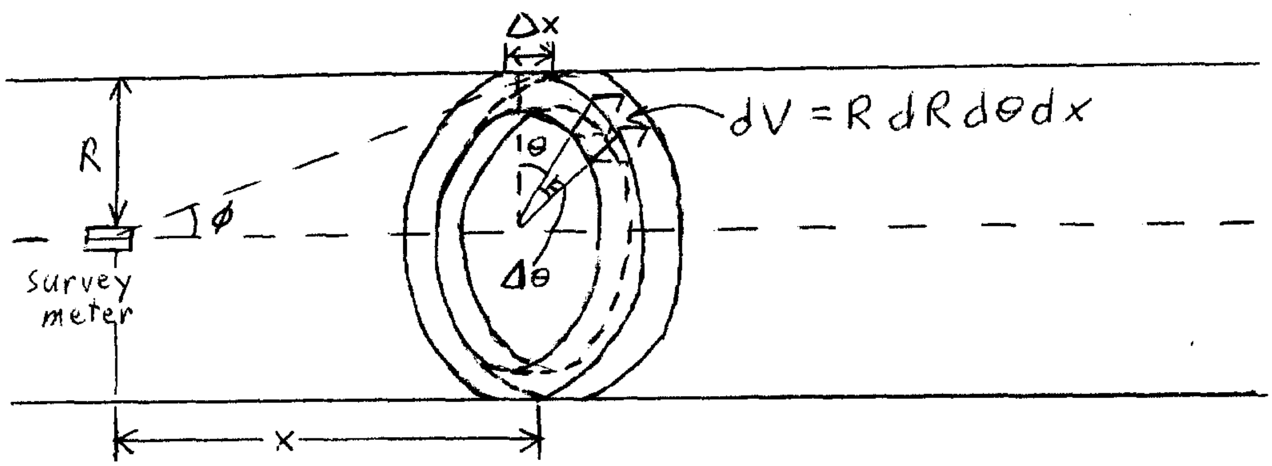
$$\int_{-\theta_{max}}^{\theta_{max}} d\theta \, B(\bar{t}) e^{-\bar{t}} = 2\int_{0}^{\theta_{max}} d\theta \, B(\bar{t}) e^{-\bar{t}} \approx 0.4 \sum_i B(\bar{t}_i) e^{-\bar{t}_j} = 1.50 \tag{5}$$

So that the ratio of total activity to the flux/($cm^2 \cdot$sec) = $1.06 x 10^7 x 1.3 = 1.38 x 10^7$.

At a photon energy of 0.75 MeV 1 mrad/hr = 725 photons/(sec$\cdot cm^2$) so that for a 20 foot main ring bending magnet 1 mrad/hr implies 725 photons/(sec$\cdot cm^2$) at the survey meter $\rightarrow 1.0 x 10^{10}$ photons/sec total source activity, therefore, 1 mrad/hr $\rightarrow$ 0.27 Ci.

Case II: Internal Survey Down the Bore of a Typical Main Ring Bending Magnet

Again several readings are made and an average is determined. Here the rectangular main ring magnet is approximated by a cylinder having an inner radius of 4.5 cm and an outer radius of 28.3 cm. (These being the average inner and outer dimensions.) For purpose of numerical integration the cylinder is divided into concentric tubes:



If one has a source of density S (stars/cm^3), without considering shielding as a function of x within the tube itself. We have the double integral (for the meter centered longitudinally in the magnet):

$$F = \int_{-\frac{L}{2}}^{\frac{L}{2}} dx \int_{o}^{2\pi} d\theta \, \frac{SR\Delta R}{4\pi(R^2+x^2)}$$

$$F = \frac{SR\Delta R}{2}\left[\frac{2}{R}\mathrm{Tan}^{-1}\frac{L}{2R}\right] = S\Delta R\mathrm{Tan}^{-1}\frac{L}{2R} \qquad (6)$$

$\mathrm{Tan}^{-1}\frac{L}{2R} = 1.55$ for the inner regions which will give the largest contribution. For cylindrically symmetric star density S(r) again obtained from Ref. 3 in the same manner as for Case I again (multiplied by 1000 for computational convenience), if t = R-4.5 cm and if we take 2 cm steps ΔR, putting in buildup and attentuation factors the total flux/($cm^2 \cdot sec$) F will be given by:

$$F = 1.55\Delta R\sum_i S(R)B(\mu t_i)e^{-\mu t_i} \tag{7}$$

$$= 1.55\Delta R\sum_i F_i$$

While the total source strength is given by:

$$S_{total} = \Delta R\sum_i 2\pi RLS(R) = \Delta R\sum_i S_i$$

Table III is a table of values involved in the sums. The factor ΔR does not appear because it is a common factor to both F and S_{total}. The innermost tube was selected with a smaller ΔR in order to improve accuracy since the reading is most sensitive to this contribution.

$$\frac{S_{total}}{F} = 5.47 \times 10^4 \tag{8}$$

Again, the selfshielding of each incremental tube must be considered. The correction will be calculated for the innermost incremental tube since it is the dominant contribution to the measured dose rate. This means that the following must be evaluated;

$$\int_0^{\frac{L}{2}} dx \; B(\mu y)e^{-\mu y}\frac{1}{x^2+R^2} \tag{9}$$

with $\phi = \mathrm{Tan}^{-1}\frac{R}{x}$, $y = \frac{t}{\sin\phi}$, $(t = 0.5)$. Integrating in 2 cm steps this becomes:

$$2\sum_i B(\mu y_i)e^{-\mu y_i}\frac{1}{x^2+R^2} \tag{10}$$

Table IV is a table of values for this sum which has the value: 0.228. Before one had the value: (from eqs. (6) and (7))

$$\frac{1.55}{2} B(\mu t)e^{-\mu t} = 0.611 \tag{11}$$

So that the "self-shielding" here gives a factor of 2.1 so that $\frac{S_{total}}{F} = 1.47 \times 10^5$.

So if 1 mrad/hr = 725 photons/(sec·cm)

1 mrad/hr $\rightarrow 1.07\text{x}10^8$ photon/sec $\rightarrow$ 2.88 mCi/(mrad/hr)

total activity.

Of course, the crucial test is measurements made with actual Fermilab magnets. Since the magnets are infinite sources, to good approximation, as seen by the meter, the results can be applied to magnets of other dimensions by use of the following formulae (as long as the length is at least a meter).

1. Outside measurement of magnet of main ring size in cross sectional area. Take average reading in mrad/hr at 1' from the side of the magnet:

   [reading in mrad/hr]x[length in feet]x[14] = total activity in mCi (12)

2. Inside measurement for magnet of main ring size in cross section taking average reading down the bore:

   [reading in mrad/hr]x[length in feet]x[0.14] = total activity in mCi (13)

For legal purposes (such as shipments of radioactive magnets) one should put in a safety factor and use the highest value if both measurements are available.

Another consideration as far as transportation requirements are concerned is that in the Department of Transportations LSA (low specific activity) category, magnets are only shipped if the specific activity is less than 0.001 mCi/gm. A standard magnet if main ring cross section has about $5.3\text{x}10^5$ gm/ft so that:

1. The average external reading should not exceed 38 mrad/hr (8 mrad/hr including a safety factor of 5).

2. The average inside reading should not exceed 3800 mrad/hr (800 mrad/hr including a safety factor of 5).

Of course, shipments off site must be made following the procedures found in the Fermilab Radiation Guide. For Fermilab beam line magnets of different cross sectional area, the results should be accurate to within approximately a factor of 2 if the numbers above are used. For somewhat better accuracy the numbers in eqs. (12) and (13) should be multiplied by $\frac{A}{390}$ where A is the cross sectional area of the desired magnet in sq. in. Of course, the accuracy will decrease for geometries greatly different than that used in this paper.

Table V is a table of measurements of typical magnets in storage at the Magnet Facility comparing consistency of internal and external measurements and calculated activities. As one can see, agreement within a factor of 2 is obtained on the average. The largest deviation may be cases where a hot spot near the beam exit end of the magnet was involved, invalidating the activation model used here.

REFERENCES

1. Accelerator Health Physics, H. W. Patterson and R. H. Thomas, and (Academic Press, 1973) p. 505.

2. P. J. Gollon, values derived from ORNL TM-3945.

3. A. Van Ginneken and M. Awschalom, High Energy Particle Interactions in Large Targets, (Fermilab), Fig VIII, 70.

4. Radiological Health Handbook, 1970 Revision, p. 145.

Table I

Table of Values for External Measurement (Case I)

$\overline{x}$	$\overline{r}$	$\mu\overline{x}$	$B(\mu x)$	$e^{-\mu\overline{x}}$	$S(R)$	$F(\overline{x})$
2	32.4	1.05	1.9	0.35	0.04	3.2×10^{-7}
6	28.6	3.15	4.44	0.043	0.06	1.2×10^{-7}
10	24.9	5.25	7.65	5.25×10^{-3}	0.085	3.3×10^{-8}
14	21.2	7.35	13.34	6.43×10^{-4}	0.15	1.1×10^{-8}
18	17.8	9.45	16.8	7.9×10^{-5}	0.24	2.5×10^{-9}
22	14.6	11.55	21.1	9.6×10^{-6}	0.4	5.6×10^{-10}
26	11.8	13.65	26.4	1.2×10^{-6}	0.6	1.3×10^{-10}
30	10.0	15.75	29.0	1.6×10^{-7}	0.8	2.3×10^{-11}
34	9.6	17.85	36.0	1.8×10^{-8}	0.85	3.3×10^{-12}
38	10.8	19.95	48	2.2×10^{-9}	0.7	4.1×10^{-13}
42	13.1	22.05	55	2.7×10^{-10}	0.5	3.9×10^{-14}
46	16.1	24.15	large	M	0.3	M
50	19.5	26.25	large	M	0.18	M
54	23.1	28.35	large	M	0.10	M
58	26.8	30.45	large	M	0.08	M
62	30.5	32.55	large	M	0.05	M
64	32.4	33.60	large	M	0.04	M
				SUMS =	5.175	4.9×10^{-7}

M denotes minimal value dominating over "large" values of $B(\mu\overline{x})$.

Table II

Correction for Self Shielding of Each Slice in Case I

θ (radian)	$\frac{\mu \bar{x}}{\cos \theta}$	$B(\bar{t})$	$xe^{-\bar{t}}$
0*	1.05	1.9	0.332
0.2	1.07	1.9	0.651
0.4	1.14	1.9	0.607
0.6	1.27	2.0	0.561
0.8	1.51	2.5	0.552
1.0	1.94	3.2	0.459
1.2	2.89	4.3	0.238
1.4	6.17	10.5	0.021
		SUM =	3.75

*An angular step $\Delta\theta = 0.1$ for 0 degree pt.

Table III

Table of Values for Internal Measurement (Case III)

R	t	μt	$B(\mu t)$	$e^{-\mu t}$	S(R)	F_i	S_i
5*	0.5	0.236	1.00	0.789	3	2.74	43122
7	2.5	1.18	1.87	0.307	2	1.77	53658
9	4.5	2.12	2.84	0.12	1	0.537	34994
11	6.5	3.06	4.14	0.047	0.9	0.27	37944
13	8.5	4.00	5.39	0.018	0.5	0.075	24913
15	10.5	4.94	6.98	7.2×10^{-3}	0.35	0.027	20123
17	12.5	5.88	8.56	2.3×10^{-3}	0.25	0.009	16289
19	14.5	6.82	10.2	1.1×10^{-3}	0.2	0.003	14564
21	16.5	7.77	12.0	4.2×10^{-4}	0.15	M	12073
23	18.5	8.71	14.0	1.6×10^{-4}	0.12	M	10578
25	20.5	9.68	16.2	6.4×10^{-5}	0.08	M	7665
27	22.5	10.6	18.0	2.5×10^{-5}	0.07	M	7243
					SUMS =	5.43	297040

*Corrected for a ΔR of 1.5 instead of 2 cm.

Table IV

Self Shielding Correction (Case II)

x	ϕ^o	y	μy	$B(\mu y)$	$e^{-\mu y}$	$\frac{1}{x^2+R^2}$	Product
1	78.6	0.51	0.24	1.0	0.786	0.038	0.029
3	59	0.58	0.27	1.2	0.761	0.029	0.026
5	45	0.707	0.32	1.3	0.717	0.02	0.018
7	35	0.86	0.41	1.4	0.666	0.013	0.012
9	29	1.03	0.49	1.43	0.615	0.009	0.0079
11	24.4	1.21	0.57	1.5	0.565	0.007	0.0059
13	21.0	1.39	0.65	1.6	0.522	0.005	0.0042
15	18.4	1.58	0.74	1.7	0.477	0.004	0.0032
17	16.4	1.77	0.84	1.8	0.431	0.003	0.0023
19	14.7	1.96	0.92	1.87	0.397	0.003	0.0048
21	13.4	2.15	1.01	1.89	0.364	0.002	0.0014
23	12.3	2.34	1.1	2.0	0.332	0.0018	0.0011
25	11.3	2.54	1.2	2.1	0.301	0.0015	0.0009
27	10.5	2.74	1.3	2.12	0.272	0.0013	0.0007
						SUM =	0.114

Table V

Results from Surveys of Main Ring Magnets

Length (ft)	External mrad/hr	mCi	Internal mrad/hr	mCi	Outside* / Inside
20	0.11	30.8	11	31.7	0.971
20	2.8	784	62	179	4.38
20	0.23	64.4	21	60.5	1.06
10	0.24	33.6	8	23	1.46
3	0.32	13.4	12	34.6	0.387
20	0.02	5.6	1	2.9	1.93
20	0.15	42	5	14.4	2.91
20	0.1	28	4	80.7	0.347
20	0.12	33.6	9	25.9	1.30
20	0.15	42	15	43.2	0.972
4	0.1	5.6	5	14.4	0.388
20	0.07	19.6	15	43.2	0.453

*Outside/Inside is the ratio of the two calculated activities with the following mean and standard deviation: Mean = 1.37, Standard Deviation = 1.20.